

Stochastic Optimization for Variable Rate Applications with Time-Varying Statistics

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Abstract—We present a scheme to ensure Quality of Service for buffered variable rate applications. The scheme does not require knowledge of the channel distributions, indeed will track changing statistics, and is effectively spectrum optimal. The scheme employs stochastic approximation where the optimal solution is learned through updates of the Lagrange multiplier. The convergence speed and nearness to optimality are found, and the buffer stability probabilities are met. Analysis and simulations are provided to validate the scheme's performance.

I. INTRODUCTION

Mobile wireless users experience a dynamic environment that makes reliable transmission difficult. As mobile applications grow more diverse, cross-layer optimization is a suggested method to maximize efficiency while meeting application requirements. One of the general objectives is to maximize the spectral efficiency for the given bandwidth while minimizing any service interruption to the user, thus maximizing both the supported users and maintaining Quality of Service (QoS).

In many applications, such as video, a mobile user utilizes a buffer to minimize service disruption and hedge against deep fades. To satisfy the user's QoS requirement, the buffer occupancy cannot underflow, or risks service interruption. In [1], a cross-layer transmission scheme was introduced that utilized a buffer state feedback bit to vary the average power and maintain buffer stability. However, the scheme required knowledge of the channel distribution.

In mobile scenarios, the complete channel statistics are largely unknown and time-varying. To maintain continual QoS support, solving a single optimization problem and using the solution for a significant duration seems sub-optimal and impractical. Rather, the optimal solution is likely to be varying, suggesting an adaptive algorithm that does not require channel statistics, yet guarantees optimality, is desirable. The proposed scheme is an adaptive stochastic approximation method [2].

The literature for variable rate applications with quality of service requirements encompasses different, and often seemingly disjoint areas. Ideas from video encoding, utility maximization, queuing theory, and effective bandwidth approach this problem in different manners spanning different layers of the OSI Model to yield cross-layer solutions. Variable rate video encoding techniques have been studied in [3], [4], and dynamic programming solutions [5], [6]. In [7], a cross-layer utility maximization framework is posed to balance rate maximization and fairness. In [8], buffer stability is examined, however the scheme requires knowledge of the

statistical distributions. Effective bandwidth function theory studies different classes of traffic, and the required bandwidth to satisfy QoS, originally developed for wired ATM networks [9]. More recently, this theory is applied to wireless channels to obtain capacity expressions with probabilistic delay constraints [10],[11].

In this paper, a cross-layer transmission scheme is suggested to provide QoS support to a user in a fading channel. By adaptively varying the rate and power, the transmitter guarantees a minimum number of bits enter the user's buffer to prevent underflow, hence continuity of the user's application. The scheme is designed to be spectrally efficient, and does not require *a priori* knowledge of the channel distribution. By utilizing stochastic approximation, the algorithm tracks the optimal solution even as channel statistics change. The paper investigates the convergence and steady-state properties of the scheme, and determines the power policy required to satisfy buffer stability.

II. SYSTEM MODEL

Consider a mobile terminal running an application that draws a random number of r_n bits from a buffer at time step n . Moreover, let the rate that bits enter the buffer, determined by the transmitter and fading channel, be R_n . Then the buffer occupancy is given as,

$$\begin{aligned} b_n &= b_{n-1} + R_n - r_n \\ &= b_0 + \sum_{i=1}^n R_i - \sum_{i=1}^n r_i \end{aligned} \quad (1)$$

Where b_0 is the initial buffer occupancy that is assumed to be set via a pre-loading scheme. This paper assumes a flow model, hence non-integer number of bits.

The application traffic r_n is modeled as a nominal rate plus an independent random variable with unknown distribution. The nominal rate is R^{app} , and the variance is σ_{app}^2 .

Channel State Information (CSI) given as γ_n are assumed to be independent and known at the transmitter, however the fading distribution $p(\gamma)$ is unknown and can be slowly time varying.

III. PROPOSED SCHEME

The goal of the proposed scheme is to transmit a minimum number of bits within a finite-time horizon and be spectrally optimal. Specifically, the scheme determines the power policy

P_n that minimizes the average transmit power $\mathbb{E}[P]$ while guaranteeing a minimum number of bits have been transmitted. As the user is expected to have a buffer, the criterion for stability is to guarantee the buffer level's worst-case probability of underflow is,

$$\max_n \text{pr}(b_n \leq 0) \leq \epsilon \quad (2)$$

A. Stochastic Approximation

The proposed scheme is a power minimization problem subject to an average rate target R^{trgt} , and it is shown to solve this problem optimally.

$$\begin{aligned} \min_{P \geq 0} \quad & \mathbb{E}[P_n] \\ \text{subject to:} \quad & \mathbb{E}[\log_2(1 + \gamma_n P_n)] = R^{\text{trgt}} \end{aligned} \quad (3)$$

Where determining R^{trgt} is the key to meeting the ϵ criterion and is found in Section III-C.

When the fading statistics are known, e.g. Rayleigh, Rician, Nakagami, this problem is solved by using the fading distribution and determining the Lagrange multiplier or the water-level. This leads to the well known water-filling power policy [12].

However, if the fading distribution $p(\gamma)$ is unknown, stochastic approximation is used to determine the optimal power policy. The solution is a three-step algorithm with negligible complexity. The first step is to estimate the channel at time step n , and solve the optimization problem using an initial Lagrange multiplier $\lambda_n = \lambda_0$. The solution is a closed form expression,

$$P_n = \left[\lambda_n - \frac{1}{\gamma_n} \right]^+ \quad (4)$$

This implies the instantaneous rate,

$$R_n = \log_2(1 + \gamma_n P_n) \quad (5)$$

The second step is calculating the stochastic subgradients,

$$\delta g_n = R^{\text{trgt}} - R_n \quad (6)$$

And finally, the Lagrange multiplier is updated,

$$\lambda_{n+1} = \lambda_n + \Delta_n \delta g_n \quad (7)$$

Where Δ_n is the step-size and dictates convergence speed and optimality as explained in Section III-B.

B. Convergence and Optimality

The optimality of this scheme relies on the convergence of λ_n to λ^* , the optimal Lagrange multiplier. As an adaptive algorithm, the step size Δ_n affects the convergence speed, tracking properties, and nearness to optimality. For example, by decreasing the step size as $\Delta_n = \Delta/n$, the solution converges to the optimal but will not track changing statistics. To ensure λ_n will adapt to a new value if the channel statistics change, a fixed step-size is required, hence this scheme assumes $\Delta_n = \Delta$. By using a fixed step-size, the solution does not converge exactly to the optimal value, rather

it converges in probability to the optimal solution and is given as [13],

$$\text{Pr}[|\lambda_n - \lambda^*| \geq \alpha | \lambda_0] \leq A_1(\Delta) + A_2(\lambda_0) \exp(-h(\Delta)n) \quad (8)$$

the term $A_1(\Delta)$ is a constant whose magnitude is a function of the step-size, and is the random cloud about the optimal solution in steady-state. The second term is the initial transient response, whose effect decreases exponentially fast in time. In steady-state, the deviation $z_n = \lambda_n - \lambda^*$ is shown to be i.i.d. $\sim \mathcal{N}(0, \sigma_z^2)$ [13, §8.4], where σ_z^2 is determined below. In essence, a small step-size takes longer to converge, but is closer to the optimal solution. A sample-path of λ_n is given in Figure 1.

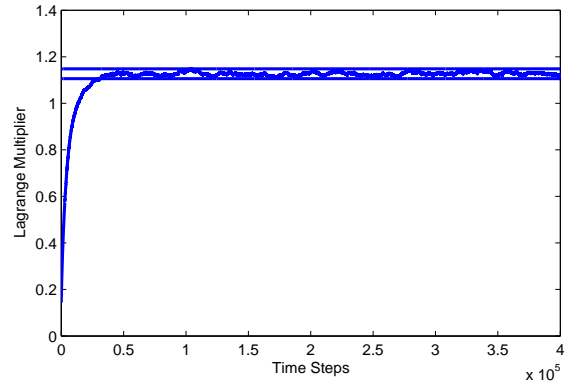


Fig. 1. λ^n converges in probability exponentially fast, $3\sigma_z$ confidence interval given.

To determine σ_z^2 , a result from stochastic approximation is required. Given an iterative expression of the form [13, Eq (8.2.1)],

$$\lambda_{n+1} = \lambda_n + \Delta[h(\lambda_n) - M_n] \quad (9)$$

where $h(\lambda_n)$ is a deterministic continuously differentiable function, and M_n is a Martingale Difference Sequence (MDS), i.e., a sequence $\{X_1, X_2, \dots, X_{n+1}\}$ is an MDS if $\mathbb{E}[X_{n+1} | X_1, \dots, X_n] = 0$. Then, the following result holds [13, Eq (8.4.1)] in steady-state,

$$2\nabla h(\lambda^*)\Delta\sigma_z^2 + \Delta^2\sigma_M^2 = 0 \quad (10)$$

Where σ_M^2 is the variance of the MDS. In order to express Eqn (7) in the form of Eqn (9), let $\zeta_n = \log_2(\gamma_n)$ and $\zeta_0^n = \log_2(1/\lambda_n)$, then from Eqn (4) and Eqn (5), the instantaneous rate is,

$$R_n = \begin{cases} \zeta_n - \zeta_0^n, & \zeta_n \geq \zeta_0^n \\ 0, & \zeta_n < \zeta_0^n \end{cases} \quad (11)$$

To satisfy Eqn (9), R_n must be written as a differentiable function, ergo consider the new random variable,

$$\tilde{\zeta}_n = \begin{cases} \zeta_n, & \zeta_n \geq \zeta_0^n \\ \zeta_0^n, & \zeta_n < \zeta_0^n \end{cases} \quad (12)$$

So that, $R_n = \tilde{\zeta}_n - \zeta_0^n$

Now, the update of λ_n is expressed as a continuously differentiable function and a MDS,

$$\begin{aligned}\lambda_{n+1} &= \lambda_n + \Delta(R^{\text{tgt}} - R_n) \\ &= \lambda_n + \Delta(R^{\text{tgt}} - \tilde{\zeta}_n + \zeta_0^n + (\mathbb{E}\tilde{\zeta} - \mathbb{E}\tilde{\zeta})) \\ &= \lambda_n + \Delta(R^{\text{tgt}} + \zeta_0^n - \mathbb{E}\tilde{\zeta} - M_n)\end{aligned}\quad (13)$$

From Eqn (9) and Eqn (13), $h(\lambda_n) = R^{\text{tgt}} + \zeta_0^n - \mathbb{E}\tilde{\zeta}$ and $M_n = \tilde{\zeta}_n - \mathbb{E}\tilde{\zeta}$. To solve Eqn (10), $\nabla h(\lambda^*)$ is given as,

$$\nabla h(\lambda^*) = -\frac{1}{\lambda^*} \log_2(e) \quad (14)$$

And $\sigma_M^2 = \tilde{\sigma}_\zeta^2$, i.e. the variance of $\tilde{\zeta}$. This leads us to the sampling noise variance,

$$\sigma_z^2 = \Delta \tilde{\sigma}_\zeta^2 \lambda^* \frac{c_1}{2} \quad (15)$$

where $c_1 = \ln(2)$.

However, a useful bound on σ_z^2 is in terms of σ_ζ^2 not $\tilde{\sigma}_\zeta^2$, because σ_ζ^2 is a only a function of the channel, while $\tilde{\sigma}_\zeta^2$ relies on the power scheme, i.e. cutoff power in poor SNR. In Section VI, the result $\sigma_z^2 \geq \tilde{\sigma}_\zeta^2$ is derived.

Hence, the stochastic approximation approach with a fixed step-size solves the optimal Lagrange multiplier within the uncertainty of z_n . And λ_n is modeled as $\mathcal{N}(\lambda^*, \Delta \sigma_\zeta^2 \lambda^* \frac{c_1}{2})$.

C. Rate Target

The rate target R^{tgt} necessary to meet the buffer probability guarantee ϵ is determined in a similar manner to [1], in which case the distribution $p(\gamma)$ was assumed to be known hence $\sigma_z^2 = 0$, and the traffic was constant hence $\sigma_{\text{app}}^2 = 0$. A complimentary approach using the Large Deviation principle and effective bandwidth theory results in a similar expression [11].

Two assumptions are made: first, the buffer size is large compared with the application's rate requirement for a given time step:

$$N = B_{\text{th}}/R^{\text{app}} > 10 \quad (16)$$

This assumption is realistic as buffer size is typically much larger than the number of bits an application uses in a scheduling instance. The second assumption is the initial buffer state can be set through an initial buffer loading transmission scheme $b_0 = B_{\text{th}}$ (e.g. the user's application does not begin drawing bits until the buffer state crosses the threshold).

The buffer underflow guarantee is,

$$\epsilon \geq 1 - \min_{n:n \geq N} \text{pr} \left(\sum_{i=1}^n R_i + B_{\text{th}} - \sum_{i=1}^n r_i > 0 \right) \quad (17)$$

The rates entering the buffer during n time steps is given as,

$$\begin{aligned}\sum_{i=1}^n R_i &= \sum_{i=1}^n \tilde{\zeta}_i - \zeta_0^i \\ &\stackrel{(a)}{\approx} \sum_{i=1}^n \tilde{\zeta}_i + \log_2(\lambda^* + z_i) \\ &\stackrel{(b)}{\approx} \sum_{i=1}^n \tilde{\zeta}_i + \sum_{i=1}^n \log_2(\lambda^*) + \frac{1}{c_1 \lambda^*} \sum_{i=1}^n z_i \\ &\stackrel{(c)}{=} W_n + n \log_2(\lambda^*)\end{aligned}\quad (18)$$

(a) Follows from Section III-B, i.e. modeling the Lagrange multiplier as a Gaussian random variable centered around optimal, (b) follows from the small angle approximation $\log_2(1+x) \approx x \log_2(e)$ which assumes a sufficiently small step-size, and (c) follows from the Central Limit Theorem as the buffer size is large from Eqn (16), and in steady-state, the R_i 's maintain the same statistics and are independent and identically distributed. Therefore, W_n is modeled as a Gaussian random variable. It follows that $W_n = \sum_{i=1}^n \left(\tilde{\zeta}_i + \frac{1}{c_1 \lambda^*} z_i \right)$, and hence $W_n \sim \mathcal{N} \left(n \mathbb{E}[\tilde{\zeta}], n \sigma_\zeta^2 \left[1 + \frac{1}{2c_1 \lambda^*} \Delta \right] \right)$.

The bits draining from the buffer during n time steps is given as $Z_n = \sum_{i=1}^n r_i$. Again, using the CLT, Z_n is modeled as a Gaussian random variable $Z_n \sim \mathcal{N}(nR^{\text{app}}, n\sigma_{\text{app}}^2)$.

To simplify notation, consider $R^{\text{tgt}} = \mathbb{E}[R_n]$,

$$R^{\text{tgt}} = \log_2(\lambda^*) + \mathbb{E}[\tilde{\zeta}] \quad (19)$$

and the total variation of the buffer,

$$\sigma_{\text{tot}}^2 = \sigma_\zeta^2 \left(1 + \frac{1}{2c_1 \lambda^*} \Delta \right) + \sigma_{\text{app}}^2 \quad (20)$$

Then, to meet the stability criterion in Eqn (17),

$$\epsilon = \max_{n:n \geq N} Q \left(\frac{B_{\text{th}} + nR^{\text{tgt}} - nR^{\text{app}}}{\sqrt{n}\sigma_{\text{tot}}} \right) \quad (21)$$

The argument of the Q -function is given as $f(n)$, then using decreasing-monotonicity of the Q -function and convexity of $f(n)$, N^* is found that minimizes $f(n)$. Therefore,

$$\begin{aligned}N^* &:= \arg \min_n \frac{B_{\text{th}} + nR^{\text{tgt}} - nR^{\text{app}}}{\sqrt{n}\sigma_{\text{tot}}} \\ &= \frac{B_{\text{th}}}{R^{\text{tgt}} - R^{\text{app}}}\end{aligned}\quad (22)$$

Applying N^* , the result from Eqn (22) into Eqn (21), the underflow guarantee is

$$\epsilon = Q \left(\frac{2}{\sigma_{\text{tot}}} \sqrt{B_{\text{th}} (R^{\text{tgt}} - R^{\text{app}})} \right) \quad (23)$$

This leads to the target rate that ensures the worst-case probability of underflow is less than ϵ , $R^{\text{tgt}} = R^{\text{app}} + \Delta R$, where

$$\Delta R = \frac{\sigma_{\text{tot}}^2 [Q^{-1}(\epsilon)]^2}{4B_{\text{th}}} \quad (24)$$

This expression has commonality with the derivation in [1], and provides a relationship between the initial buffer level, the QoS guarantee, and the uncertainty in the system. As intuition suggests, more uncertainty requires a larger rate deviation to maintain buffer stability. In this case, the variance is a function of the channel variation σ_C^2 , the step-size Δ , and the traffic variation σ_{app}^2 .

IV. NUMERICAL RESULTS: BUFFER STABILITY SCHEME

In this section, the numerical results are presented by implementing the proposed scheme for both an underflow guarantee *and* an overflow guarantee. Rather than guaranteeing a minimum number of bits is being transmitted, this section implements a window guarantee, i.e. the transmitted bits is above the minimum number, and below a maximum number to ensure that a buffer does not underflow or overflow. The scheme is implemented with two rate targets, R_+^{tgt} and R_-^{tgt} , and depending on the instantaneous buffer level b_n switches between the solutions of the two rate targets, as given in the state diagram Figure 2.

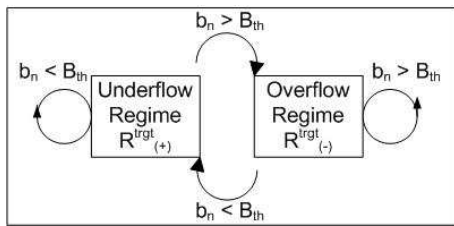


Fig. 2. Switching scheme based on instantaneous buffer level.

A sample path of the Lagrange multipliers used in this switching scheme is given in Figure 3.

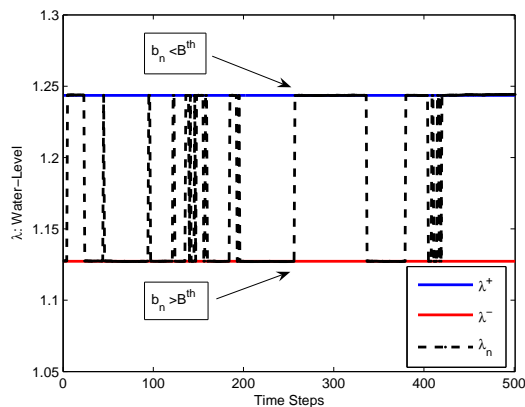


Fig. 3. Proposed scheme switches between solutions of two different optimization problems based on the instantaneous buffer level.

In Figure 4, the proposed scheme is compared to single-level water filling (SWF), a spectrally optimal transmission scheme for an average rate target. As is clear, the proposed scheme is able to maintain buffer stability and keep the buffer levels within a finite window, while SWF cannot.

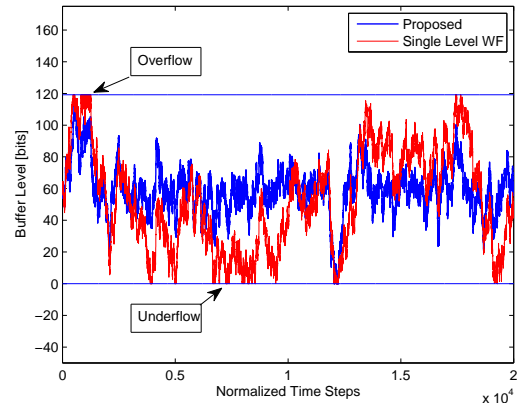


Fig. 4. Proposed scheme provides buffer stability, while single-level water-filling does not provide overflow/underflow guarantees.

A. Spectral Efficiency

The proposed algorithm is stable and optimal regardless of fading distribution, and indeed, does not need to know the distribution *a priori*. However, spectral efficiency is a function of the fading statistics, hence to gain insight into efficiency, Rayleigh and Nakagami ($m = 2$) are considered. The spectral efficiency of the proposed scheme is compared to the Single-Level Water-Filling (SWF), optimal truncated channel inversion (TCI) [12], and the Buffer State Information (BSI) scheme [1]. Note that SWF, TCI, and BSI require the channel distribution $p(\gamma)$, while the proposed scheme does not. SWF maintains the same long-term average rate, but does not guarantee buffer stability. Nonetheless, SWF serves as an upper-bound for the optimal scheme.

The QoS parameter $\epsilon = 5 \times 10^{-3}$. To make comparisons with Shannon capacity, the applications average rate requirement R^{app} is set to the ergodic capacity C , and then only the normalized buffer size, $N = B_{th}/R^{app}$, a measure of delay, needs to be specified.

The spectral efficiency curves measure the efficiency of a transmission scheme normalized by the allocated bandwidth. As can be seen, the proposed scheme has effectively the same spectral efficiency as SWF and BSI over a wide range of SNR values, implying near optimal spectral efficiency. In Figure 5 and Figure 6, $N = 10$, the minimum value that satisfies assumption Eqn (16). At higher N , the proposed scheme converges to the efficiency of SWF.

V. SUMMARY

A scheme is proposed that ensures buffer stability, does not require knowledge of the channel statistics, and is effectively spectrum optimal. By using ideas from stochastic approximation, the algorithm learns and tracks the optimal solution, and the convergence speed and nearness to optimality are given. The scheme's performance is compared to the well-known water-filling and truncated channel-inversion transmission schemes, where only the proposed scheme is able

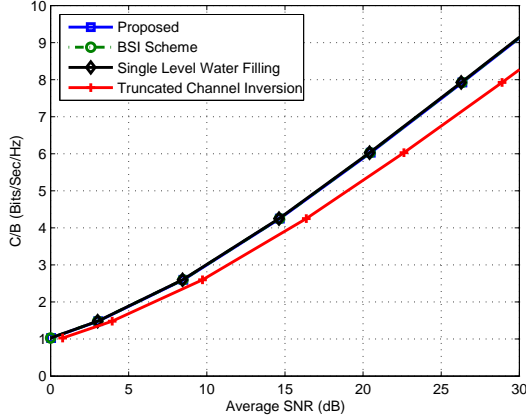


Fig. 5. Spectral efficiency for Rayleigh Fading.

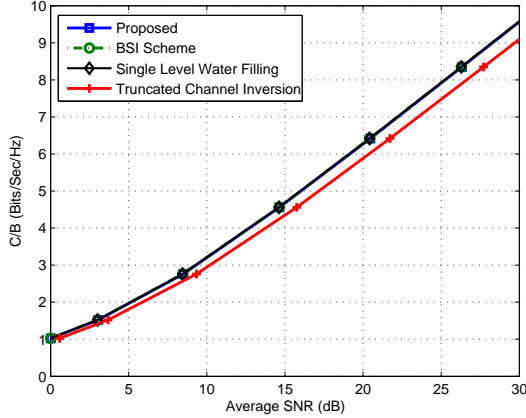


Fig. 6. Spectral efficiency for Nakagami-m Fading (m=2).

to maintain buffer stability *and* have nearly optimal spectral efficiency. Analysis and simulations are provided as validation.

VI. APPENDIX

To bound the variance of the rates achieved by water-filling in a fading environment, consider the random variable $R = \log_2(\gamma)$ where $p_\gamma(x)$ is the pdf of a positive random variable. And \tilde{R} is a random variable with the water-filling structure,

$$\tilde{R} = \begin{cases} \log_2(\gamma), & \gamma \geq 1 \\ 0, & \gamma < 1 \end{cases}$$

The variance of R is,

$$\sigma_R^2 = \int_0^\infty \log_2^2(x)p_\gamma(x)dx - \left(\int_0^\infty \log_2(x)p_\gamma(x)dx \right)^2$$

and the variance of \tilde{R} is

$$\tilde{\sigma}_R^2 = \int_1^\infty \log_2^2(x)p_\gamma(x)dx - \left(\int_1^\infty \log_2(x)p_\gamma(x)dx \right)^2$$

To show $\sigma_R^2 \geq \tilde{\sigma}_R^2$, consider $\sigma_R^2 - \tilde{\sigma}_R^2$. With some manipulation,

$$\begin{aligned} \sigma_R^2 - \tilde{\sigma}_R^2 &= \int_0^1 \log_2^2(x)p_\gamma(x)dx - \left(\int_0^1 \log_2(x)p_\gamma(x)dx \right)^2 \\ &\quad - 2 \left(\int_0^1 \log_2(x)p_\gamma(x)dx \right) \left(\int_1^\infty \log_2(x)p_\gamma(x)dx \right) \\ &\stackrel{(a)}{\geq} -2 \left(\int_0^1 \log_2(x)p_\gamma(x)dx \right) \left(\int_1^\infty \log_2(x)p_\gamma(x)dx \right) \\ &\stackrel{(b)}{\geq} 0 \end{aligned}$$

Where (a) follows from the Cauchy-Schwarz Inequality, and (b) follows from negativity of the first integral, and positivity of the second integral. Thus, the variation of the rates achieved by water-filling is upper-bounded by an expression dependent only on the channel.

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