Mobility Dependent Feedback Scheme for point-to-point MIMO Systems

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***Abstract*—A MIMO diversity scheme that utilizes varying amounts of channel state information (CSI) for mobile users is presented. CSI at the transmitter is obtained through a time-duplexed feedback channel, and thus by varying the periodicity of feedback intervals, an optimal balance is struck between obtaining accurate CSI and minimizing the overhead of uplink transmission. The optimal feedback amount is shown to have a strong dependence on the user's average SNR and Doppler spread. The proposed solution optimally switches between beamforming and Orthogonal Space-Time Block Coding and has negligible loss of performance with respect to more complex optimal schemes.**

***Index Terms*—MIMO diversity, channel state information, fixed rate, feedback**

I. Introduction

Current and future wireless technologies are required to support high-data rate applications with stringent Quality of Service (QoS) constraints. Applications such as VoIP and streaming audio/video require a reliable connection that experiences limited outage. In Metropolitan Area Networks (MAN) such as WiMAX, users are expected to have widely varying channel conditions and mobilities that cause deep fades and make reliable transmission difficult. Diversity techniques combat fading by having redundant information sent in time, frequency, and/or spatial dimensions. If perfect Channel State Information at the Transmitter (CSIT) is present, the optimal transmit scheme to minimize probability of error is to apply beamforming [1]. If CSIT is unknown, an Orthogonal Space-Time Block Coding (OSTBC) scheme can be employed [2]. As intuition suggests, with additional information, beamforming outperforms OSTBC by achieving an array gain, but requires a costly feedback mechanism to obtain channel knowledge [3]. In [4] Jöngren proposed a scheme that computes the optimal linear combination between beamforming and OSTBC based on the quality of the channel estimate. While an optimal scheme, for a fast moving channel this scheme would become prohibitively complex since for each transmitted symbol a convex optimization problem must be solved.

We consider a point-to-point MIMO system where a Base Station (BS) is transmitting a fixed rate to a user with varying mobility, e.g. stationary, pedestrian, vehicular. The transmit objective is to minimize the probability of error through either a closed-loop scheme such as beamforming, or an open-loop scheme like OSTBC. For the closed-loop scheme, CSIT is obtained through a time-duplexed feedback channel.

In this paper, two main ideas are presented:

1) An analytical bound on the optimal rate of CSI updates is found for a mobile user with a fixed rate constraint. This analysis shows how the optimal rate of CSI updates will have a strong dependence on the mobility profile of the user and on its SNR.

2) A scheme is proposed that attains nearly identical error performance to an optimal scheme [4] with reduced complexity. The proposed scheme chooses either beamforming or OSTBC based on mobility and average SNR, while optimally varying CSIT updates. Through a mix of simulation and analysis, we show both schemes offering the same performance.

The organization of the paper is as follows: Section II formalizes the system model for the proposed scenario, in Section III we present a theoretical analysis of OSTBC and beamforming, and derive a bound on the optimal amount of feedback, in Section IV we propose a simple transmission scheme derived from Section III. Finally, in Section V we present simulation results supporting the validity of the model assumed.

II. System Model

Consider a point-to-point MIMO system as shown in Figure 1, where the transmitter and receiver have M_T and M_R antennas, respectively. The channel matrix $\mathbf{H}$ is $M_R \times M_T$. The individual complex gains h_{ij} follow a zero-mean complex Gaussian distribution with variance σ_h^2.The data symbols $\mathbf{S}$ are coded via an OSTBC encoder, and the coded symbols are given as $\mathbf{C}$ of size $M_T \times R$, where R is the time duration of one block. The coded symbols pass through a linear transformation $\mathbf{W}$ [4] and transmitted through the channel $\mathbf{H}$. At the receiver, the received symbols are corrupted with noise $\mathbf{N}$ of size $M_R \times R$. The noise is assumed to be i.i.d. zero-mean complex Gaussian noise with covariance matrix $\sigma_N^2 \mathbf{I}_{M_R}$. With this model, the received signal $\mathbf{Y}$ is $M_R \times R$ is given as

$$\mathbf{Y} = \mathbf{HWC} + \mathbf{N} \quad (1)$$

The received signal is decoded with an optimal Maximum-Likelihood (ML) detector. CSI is assumed to be known perfectly at the receiver, and fed back via a TDD scheme to the transmitter. Thus, the total transmission time is divided between t_{UL} uplink and t_{DL} downlink seconds, respectively.

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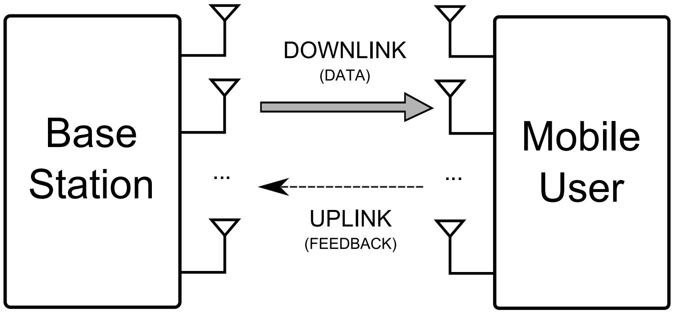

Fig. 1. Point-to-point MIMO system.

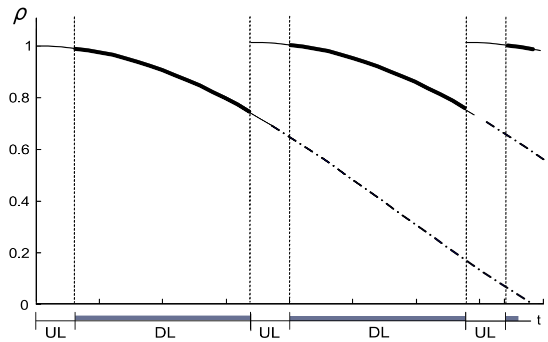

Fig. 2. Evolution of the correlation coefficient between the CSI and the actual channel.

We define the feedback fraction β as the fraction of UL transmission time to the total transmission time:

$$\beta = \frac{t_{\rm UL}}{t_{\rm UL} + t_{\rm DL}} \tag{2}$$

In our model, CSI is perfectly described in α bits by quantizing each complex term h_{ij} to $\alpha/(M_T M_R)$ bits and fed back to the transmitter at a constant rate $R_{\rm UL}$, thus $t_{\rm UL} = \alpha/R_{\rm UL}$ is constant.

The downlink has to support a fixed average rate $R_{\rm DL}$ regardless of the channel time used for the uplink. This can be achieved by coding/constellation adaptation on the downlink transmission. For simplicity we will assume uncoded QAM transmission, though the results can be extrapolated to coded systems. If we define N_0 as the nominal constellation size required to meet the rate requirement $R_{\rm DL}$ when $\beta = 0$, i.e. when there is no UL feedback, then, for $0 < \beta < 1$, the increased QAM constellation size N required to compensate for the lost DL time during feedback is $N = N_0^{1/(1-\beta)}$. Additionally for a QAM constellation we have that the minimum distance between transmitted symbols is

$$d_{min}^2 = \frac{3}{N-1} = \frac{3}{N_0^{1/(1-\beta)} - 1} \tag{3}$$

The transmitter has a delayed estimation of the channel that decorrelates from the true channel values. For the channel variation in time we will consider Jakes model [5], that is, the channel correlation function is given by the zero-order Bessel function of the first kind $\rho(\tau) = J_o(2\pi f_D \tau)$ where f_D is the doppler spread of the channel, and τ is the time delay. The estimated channel $\hat{\mathbf{H}}$ is correlated with the actual channel, $\rho(\tau) = E[h_{ij}(\tau)\hat{h}_{ij}^*]/\sigma_h^2$, where the value of $\rho(\tau)$ depends strongly on the time required to feedback the CSI and on doppler frequency. Therefore, the channel estimate is modeled as

$$\hat{\mathbf{H}}(\tau) = \rho(\tau)\mathbf{H}_o + \sqrt{1-\rho(\tau)^2}\mathbf{H}_w(\tau) \tag{4}$$

where $\mathbf{H}_o$ and $\mathbf{H}_w(\tau)$ are the actual channel and a spatially white estimation error, respectively. To clarify the model Figure 2 shows an example of the evolution of the correlation coeffient between the CSI $\hat{\mathbf{H}}(\tau)$ and the actual channel $\mathbf{H}_o$. Aditionally, the transmitter is assumed to know the long-term channel statistics.

III. Analysis

The objective is to minimize average probability of error $P_{\rm e}$ for a fixed rate application by varying the rate of CSIT updates. In this section we will upper bound the probability of error obtained by the scheme in [4] with the minimum of the probability of error of the two basic diversity schemes: naive beamforming and OSTBC. Here we define naive beamforming as the scheme that assumes (often incorrectly) that the CSIT is perfect, and therefore aligns its transmitted signal with the input singular vector associated to the largest singular value of the channel estimate. To keep the tractability of the problem we will consider a $M \times 1$ MISO system with Rayleigh fading.

In this scenario there exists an optimal feedback percentage β^{opt} to minimize $P_{\rm e}$. To understand the trade-off in β, consider a high β that ensures nearly perfect CSIT, and minimizes any misalignment during beamforming, and therefore reduces $P_{\rm e}$. However, a high β requires excessive CSIT updates through a feedback channel that cannibalizes the useful DL time $t_{\rm DL}$. For a fixed rate application, any reduction in $t_{\rm DL}$ implies a necessary increase in the constellation size/coding rate, and thus a degradation in the error performance for fixed bandwidth and power constraints.

The probability of error for the OSTBC scheme for a fixed $\mathbf{h}$ averaged over all the transmission symbols can be upper bounded by the union of events bound as

$$P_e^{OSTBC} \le \bar{N}_e Q\left\{\sqrt{\eta d_{min}^2 \|\mathbf{h}\|^2}\right\} \tag{5}$$

where $\bar{N}_e$ represents the average number of neighbor points/codewords and d_{min} is the minimum distance of the constellation/coding scheme. $\eta = E_s/\sigma_N^2$ is the average received SNR on a single receive antenna. For ease of notation we introduce $k = \eta d_{min}^2/2$. By applying the Chernoff-bound and using that $\sqrt{2}\|\mathbf{h}\|$ is a χ-distributed random variable, we obtain the following upper-bound on $\bar{P}_e^{OSTBC} = E_{\mathbf{h}}\left[P_e^{OSTBC}\right]$ [1]

$$\bar{P}_e^{OSTBC} \le \frac{\bar{N}_e}{2}\left(1 + \frac{k}{M}\right)^{-M} \tag{6}$$

Note that $\bar{P}_e^{OSTBC}$ is not a function of the quality of the channel state information at the transmitter (given in our model by ρ) since OSTBC does not require CSIT.

On the other hand, the probability of error for a naive beamforming scheme with imperfect channel knowledge can be obtained from the channel model given in Section II. In order to get a closed-form expression, this probability of error is upper-bounded by the union of events bound and subsequently by Chernoff-bound as

$$\begin{aligned} P_e^{BF} &\leq \bar{N}_e Q\left\{\sqrt{\eta d_{min}^2 \langle \mathbf{h}, \hat{\mathbf{h}} \rangle}\right\} \\ &\leq \frac{\bar{N}_e}{2} \exp\left\{-k\langle \mathbf{h}, \hat{\mathbf{h}} \rangle\right\} \\ &= \frac{\bar{N}_e}{2} \exp\left\{-k\left(\rho\|\mathbf{h}\|^2 + \bar{\rho}\langle \mathbf{h}, \mathbf{h}_w \rangle\right)\right\} \end{aligned} \tag{7}$$

where we define $\bar{\rho} = \sqrt{1-\rho^2}$. By taking the expectation over the distributions of $\mathbf{h}$ and $\mathbf{h}_w$ we obtain a bound on the average probability of error

$$\begin{aligned} \bar{P}_e^{BF} &\leq E\left[\frac{\bar{N}_e}{2} \exp\left\{-k(\rho\|\mathbf{h}\|^2 + \bar{\rho}\langle \mathbf{h}, \mathbf{h}_w \rangle)\right\}\right] \\ &= \frac{\bar{N}_e}{2} E_{\mathbf{h}}\left[\exp\left\{-k\rho\|\mathbf{h}\|^2\right\} \cdot \right. \\ &\qquad \left. E_{\mathbf{h}_w|\mathbf{h}}\left[\exp\left\{-k\bar{\rho}\langle \mathbf{h}, \mathbf{h}_w \rangle\right\}\right]\right] \end{aligned} \tag{8}$$

We decompose the inner product as $\langle \mathbf{h}, \mathbf{h}_w \rangle = \|\mathbf{h}\|\|\mathbf{h}_w\|\cos(\alpha)$. Using that the individual entries of the vectors $\mathbf{h}$ and $\mathbf{h}_w$ are independent complex gaussian random variables with unit variance, both $\sqrt{2}\|\mathbf{h}\|$ and $\sqrt{2}\|\mathbf{h}_w\|$ are χ-distributed with $2M$ degrees of freedom, and the angle between both vectors α follows a $U(-\pi, \pi)$:

$$\begin{aligned} &E_{\mathbf{h}_w|\mathbf{h}}\left[\exp\left\{-k\bar{\rho}\langle \mathbf{h}, \mathbf{h}_w \rangle\right\}\right] \\ &\quad = E_{\|\mathbf{h}_w\|}\left[E_\alpha\left[\exp\left\{-k\bar{\rho}\|\mathbf{h}\|\|\mathbf{h}_w\|\cos(\alpha)\right\}\right]\right] \\ &\quad = E_{\|\mathbf{h}_w\|}\left[I_0\left(-k\bar{\rho}\|\mathbf{h}\|\|\mathbf{h}_w\|\right)\right] \end{aligned} \tag{9}$$

where I_0 is the modified Bessel function of first kind. Note that for M=1 $E_{\|\mathbf{h}_w\|}\left[I_0\left(-k\bar{\rho}\|\mathbf{h}\|\|\mathbf{h}_w\|\right)\right]$ can be written as a Marcum Q-function that can be upper bounded by $\exp\left\{1/4(k\bar{\rho}\|\mathbf{h}\|)^2\right\}$ [6]. By applying a similar bound for general M [7] we obtain a bound tight for $\bar{\rho} \to 0$:

$$E_{\|\mathbf{h}_w\|}\left[I_0\left(-k\bar{\rho}\|\mathbf{h}\|\|\mathbf{h}_w\|\right)\right] \leq \exp\left\{\frac{M}{4}(k\bar{\rho}\|\mathbf{h}\|)^2\right\} \tag{10}$$

Since this inequality holds for all $\|\mathbf{h}\|$,

$$\begin{aligned} \bar{P}_e^{BF} &\leq \frac{\bar{N}_e}{2} E_{\mathbf{h}}\left[\exp\left\{-k\rho\|\mathbf{h}\|^2\right\} \cdot \right. \\ &\qquad \left. E_{\|\mathbf{h}_w\|}\left[I_0\left(-k\bar{\rho}\|\mathbf{h}\|\|\mathbf{h}_w\|\right)\right]\right] \\ &\leq \frac{\bar{N}_e}{2} E_{\mathbf{h}}\left[\exp\left\{-\|\mathbf{h}\|^2\left(\rho k - \frac{M}{4}(\bar{\rho}k)^2\right)\right\}\right] \\ &= \frac{\bar{N}_e}{2}\left(1 + \rho k - \bar{\rho}^2 k^2 \frac{M}{4}\right)^{-M} \end{aligned} \tag{11}$$

where for the last equality we have used that the random variable $\sqrt{2}\|\mathbf{h}\|$ is χ-distributed with 2M degrees of freedom.

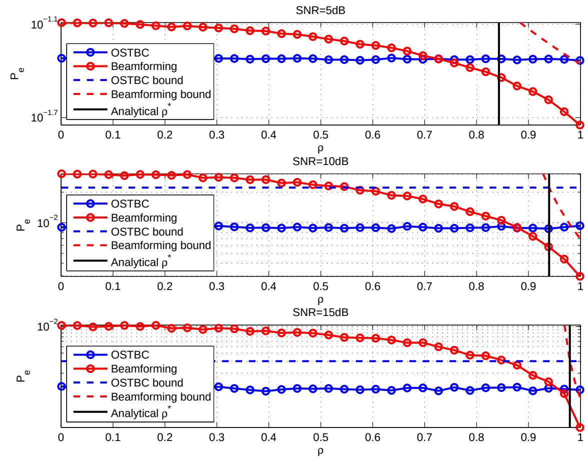

Fig. 3. Comparison between the analytical expression obtained for ρ and the actual crossing point OSTBC and beamforming curves.

Brehler showed in [8] that the asymptotic gap between the pairwise error probability with coherent detection and its Chernoff bound is a constant function of the number of antennas. Moreover, the gap vanishes for large number of antennas and the expressions (6) and (11) are asymptotically tight for high SNR, $\rho \to 1$, and increasing number of antennas. By equating (6) and (11) we obtain an approximate closed expression for the ideal switching point ρ^* between both schemes.

$$\rho^* \approx \frac{-1 + \sqrt{1 + Mk\left(\frac{Mk}{4} + \frac{1}{M}\right)}}{\frac{Mk}{2}} \tag{12}$$

Since the two upper bounds may not intersect in the same point where the actual pairwise error probabilities do, we have to take this result with care. Note that because of the asymptotically tightness of the Chernoff bound (12) turns to be a good approximation for high SNR and large number of transmit antennas, if ρ^* is close to 1 . It can be expected that the approximation still holds for moderated SNRs and number of antennas. Figure 3 shows the behavior of the approximated ρ^* with respect to the real crossing point for a set of simulations with $M_T = 2$, $M_R = 1$ and a set of different SNRs. We can see how the gap between the actual average probability of error and the bounds obtained in the previous section decreases with the SNR, and the approximation approaches the actual crossing point. Note that the general behavior of the actual ρ^* is well approximated by (12).

In the previous analysis we obtained that for $\rho > \rho^*$ naive beamforming performs better than a simple OSTBC scheme. This gives us a lower bound for the amount of needed feedback in order to use naive beamforming instead of OSTBC. To obtain some improvement the CSIT update rate has to be high enough to guarantee $\rho > \rho^*$. From (2), by assuming the worst case correlation (at the end of the downlink period) $\rho = J_o(2\pi f_D t_{\mathrm{DL}})$, the optimal feedback duration β^{opt} is lower bounded by

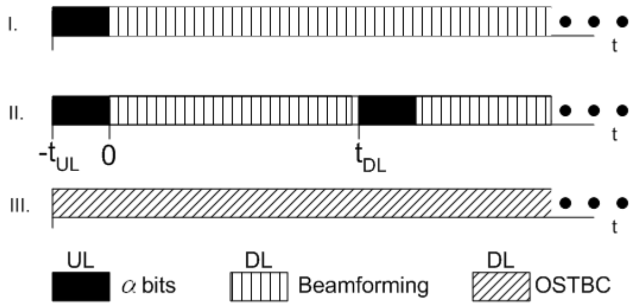

Fig. 4. Proposed scheme: I. No motion: feedback once then use beamforming; II. Small motion: increase rate of feedback, and use beamforming; III. Fast motion: no feedback and use OSTBC.

Parameter	Value
α	64 bits
SNR	5dB, 10dB, 15dB
f_D	0-250 Hz
M_T	2
M_R	1
R_{uplink}	250 kbps
$R_{downlink}$	1 Mbps
N_0	2
N_e	1

TABLE I
SIMULATION PARAMETERS.

$$\beta^{opt} \geq \frac{2\pi f_D t_{\mathrm{UL}}}{2\pi f_D t_{\mathrm{UL}} + J_o^{-1}(\rho^*)} \tag{13}$$

While the J_o is not strictly invertible for all ρ^*, it is for $\rho^* > 0.5$ and therefore is a well-defined expression for all reasonable η and d_{min}.

IV. Proposed Scheme

In the previous section we showed how the feedback is only useful for the naive beamforming scheme when $\rho > \rho^*$, where ρ^* can be approximated by (12).

If we examine the values of ρ^* for this setup, we can see that they are over 0.75 for the SNR=5dB and even higher for the remaining SNRs. Analyzing the behavior of the scheme presented in [4] for these system parameters, we realize that the scheme is working in its asymptotic regime for this value of ρ and thus behaving as a pure beamforming scheme. That is, for this level of channel quality, the optimal scheme is nearly identical to simply using the naive beamforming scheme.

Based on this observation, we propose a simplified scheme for our fixed rate communication system. The idea of this scheme is presented in Figure 4. In scenario-I, the channel is completely static and ρ equals 1, the optimal scheme is beamforming. As the user's mobility increases in scenario-II, the channel estimate starts to decorrelate from the true value, and more feedback is required, i.e. β^{opt} increases enough to guarantee that naive beamforming outperforms the OSTBC modulation. At a critical speed, the channel estimate has decorrelated beyond use as $\rho < \rho^*$ and the optimal scheme is OSTBC with $\beta^{opt} = 0$, as in scenario-III.

The performance of this very simple scheme will be compared in Section V to the optimal scheme, and to pure OSTBC and naive beamforming schemes.

V. Simulation results

In this section, two main results are presented. First, the behavior of the optimal feedback time fraction β^{opt} is shown as the user's mobility changes for different SNRs. Secondly, our scheme is compared with the performance of the scheme in [4] and the static diversity schemes of beamforming and OSTBC.

During the theoretical analysis developed in Section III we used a simplified model where the optimal scheme was approximated by two simple transmission schemes: OSTBC and naive beamforming. Moreover, worst case correlation at the end of the block was assumed to establish the lower bound on β^* (13). In the simulation we will consider the original system described in Section II where each individual space-time symbol **C** is prefiltered by the optimal scheme of [4] for each correlation value during the downlink period. The parameters of the simulation are presented in Table I.

Figure 5 shows how the optimal amount of feedback fraction β^{opt} varies with the doppler spread and thus with the mobility profile of the user. We can observe that the theoretical result obtained in Section III matches the behaviour of the more complex simulated system model. To maintain the tractability of the problem, the changes of minimum distance of the constellation were not taken into account in the theoretical analysis, and therefore the dropping behaviour of the curve has not been characterized.

For a fixed SNR, the figure shows two regions as a function of f_D: an increasing β^{opt} followed by a β^{opt} falling to zero. Both of these regions have an intuitive meaning. The amount of feedback required to minimize P_{e} initially increases with the velocity of the user, as more frequent channel estimates are needed. However, at a critical speed, i.e. the inflection point, the penalty on probability of error imposed by the increased constellation size cancels the benefit of beamforming, and therefore OSTBC is used.

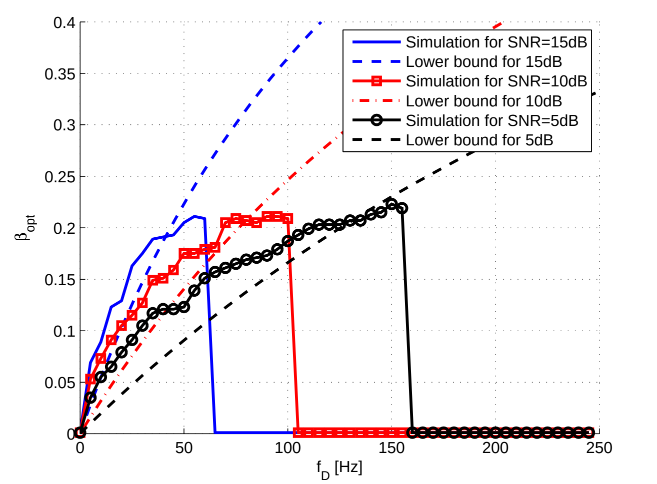

Fig. 5. The optimal fraction of time β^{opt} against the mobility of the users.

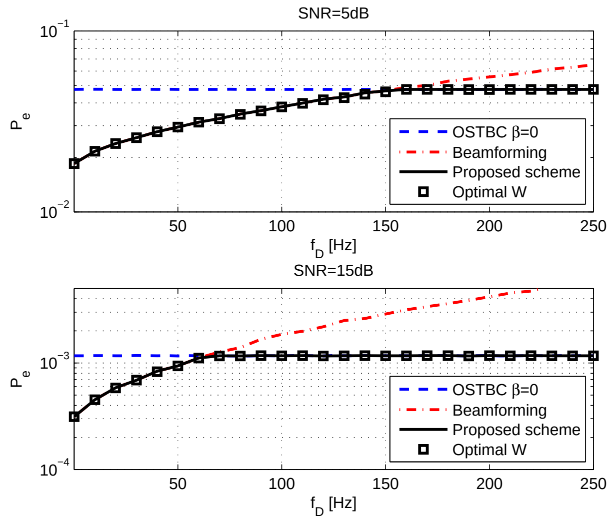

Fig. 6. Minimum probability of error for the different schemes.

At first it may seem surprising that for a fixed doppler spread the amount of useful feedback increases with the SNR. An intuitive explanation comes from the fact that for high SNR only high quality CSI is useful, since the beamformer has to be perfectly aligned with the largest mode of the channel. In other case, the cost (in terms of increased probability of error due to reduced minimum distance between constellation points) is larger than the improvement obtained by beamforming. Note, that we could have derived this behavior from equation (12): if the SNR $\eta \to \infty$ we have that $\rho^* \to 1$ and the slope of the feedback fraction approaches ∞.

If we look at the dropping point of the optimal feedback fraction β^{opt} as a function of the SNR, we can see that the feedback only helps to fast moving users with low SNR. On the other hand, users with high SNR use the OSBC scheme already at moderated speeds not requiring any feedback. The explanation of this behaviour is closely related to the higher quality of CSI required for users with better SNR. Since users with higher SNR require more feedback updates to obtain the required quality of CSI, they exceed the limit on the channel resources that can be used for feedback before other users that require less CSI updates.

Finally Figure 6 shows the performance comparison of different schemes. We can see how the simplified scheme presented in Section IV performs nearly identically to the optimal solution in [4], and outperforms the static beamforming and OSTBC schemes in some range of parameters. As we commented in Section IV, the optimal linear precoding scheme offers no important improvement with respect to the simplified scheme because at β^{opt} the optimal linear precoding is working close to its asymptotic regime, and thus can be readily approximated by its asymptotic behaviour: beamforming and OSTBC.

VI. Conclusions

In this paper, the value of the feedback was investigated for a point-to-point MIMO fixed-rate scenario. The optimal amount of feedback was shown to be a strong function of the user's mobility and average SNR. Thus, to optimally minimize probability of error, a transmitter should evaluate both user's mobility and average SNR to determine the appropriate feedback and diversity scheme. Aditionally, a simple multiplexing scheme was presented that chooses between OSTBC and Beamforming and controls the frequency of feedback depending on the user mobility profile. Under the scenario of point-to-point fixed-rate communication the proposed scheme has a negligible loss of performance with respect to the optimal scheme proposed in [4].

References

[1] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge University Press, 2003.

[2] S. Sandhu and A. Paulraj, "Union bound on error probability of linear space-time block codes," *Acoustics, Speech, and Signal Processing, 2001. Proceedings. (ICASSP '01). 2001 IEEE International Conference on*, vol. 4, pp. 2473–2476 vol.4, 2001.

[3] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, "Multiuser mimo downlink made practical: Achievable rates with simple channel state estimation and feedback schemes," *CoRR*, vol. abs/0711.2642, 2007.

[4] G. Jöngren, M. Skoglund, and B. Ottersten, "Combining beamforming and orthogonal space-time block coding," *Information Theory, IEEE Transactions on*, vol. 48, no. 3, pp. 611–627, Mar 2002.

[5] W. C. Jakes and D. C. Cox, *Microwave Mobile Communications*. Wiley-IEEE Press, 1994. [Online]. Available: http://portal.acm.org/citation.cfm?id=561302

[6] M. K. Simon and M.-S. Alouini, "Exponential-type bounds on the generalized marcum q-function with application to error probability analysis over fading channels," *Communications, IEEE Transactions on*, vol. 48, no. 3, pp. 359–366, Mar 2000.

[7] I. S. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. Academic Press, 2000.

[8] M. Brehler and M. K. Varanasi, "Asymptotic error probability analysis of quadratic receivers in rayleigh-fading channels with applications to a unified analysis of coherent and noncoherent space-time receivers," *IEEE Transactions on Information Theory*, vol. 47, no. 6, 2001.